Lecture 2: Numerical Systems

Basics of Numerical system:

1. Each numerical system is named according to its base number (Base: B) which is used in its calculations.
2. The numbers of system in each system are (0, 1, 2 … B-1).
3. Symbol in the further most right rank in any system is called (**L**east **S**ignificant **D**igit) abbreviated as **(LSD)**, and symbol in the further most left rank is called (**M**ost **S**ignificant **D**igit) abbreviated as **(MSD)**.

Types of Numerical Systems:

1. Decimal Numbers (B = 10).
2. Binary Numbers (B = 2).
3. Octal Numbers (B =8).
4. Hexadecimal Numbers (B = 16).

Decimal System: it is the same as the one we use in our lives, each decimal rank weight 10 times its right neighbor in this way the weights of the ranks from **right** to **left** is 1, 10, 100, 1000 and so on.

- Base = 10.
- The symbols used are: (0, 1, 2, 3, 4, 5, 6, 7, 8, 9).

Ex. 128

(1*100) + (2*10) + (8*1) = 100+20+8= 128

Where the base of this system is (10), therefore we can put ranks digits from **right** to **left** to represent the power of the number or the base 10, and starting from **$1=10^0$** as follow:

	1		2		8
	$1*10^2$		$2*10^1$		$8*10^0$
$(128)_{10}$ =	100	+	20	+	8

In case of fractional numbers become as follow:

10^2 10^1 10^0 . 10^{-1} 10^{-2} 10^{-3}

Binary System: Computer uses binary system, its base is 2 and it contain only 0 or 1(on or off). In binary system each rank weight twice its **right** neighbor, in this way rank weights from **right** to **left** is 1, 2, 4, 8, 16 and so on.

- Base = 2.
- The symbols used are: (0, 1).

The same thing in this system:

2^4	2^3	2^2	2^1	2^0
16	8	4	2	1

Ex. 11001

1	1	0	0	1
2^4	2^3	2^2	2^1	2^0

- **Bit:** It is the abbreviation of the two words *(**Bi**nary Digi**t**)*, it means 0 or 1, and it's the basic unit in processing the data because all the data are converted to **bits** in digital computers.

  Bit = 0 or 1

  Byte = 8 bits

  Kilobyte = 1024 byte

  Megabyte = 1024 Kilobyte

  Gigabyte = 1024 Megabyte

  Terabyte = 1024 Gigabyte

- **Decimal-to-Binary Conversion:** *(Repeated Division–by–2 Method)*

  Ex. $(14)_{10}$

  14 ÷ 2 = 7 0 (***LSD)***

  7 ÷ 2 = 3 1

  3 ÷ 2 = 1 1

  1 ÷ 2 = 0 1 (***MSD)***

  So $(14)_{10} = (1110)_2$

  Ex. Convert the decimal number $(25)_{10}$ *to binary.*

  25 ÷ 2 = 12 1 ***(LSD)***

  12 ÷ 2 = 6 0

  6 ÷ 2 = 3 0

  3 ÷ 2 = 1 1

  1 ÷ 2 = 0 1 ***(MSD)***

  So $(25)_{10} = (11001)_2$

Binary-to-Decimal Conversion:

Ex. $(1101001)_2$

2^6	2^5	2^4	2^3	2^2	2^1	2^0
1	1	0	1	0	0	1

$(1*2^6)+(1*2^5)+(0*2^4)+(1*2^3)+(0*2^2)+(0*2^1)+(1*2^0)$

$64 + 32 + 0 + 8 + 0 + 0 + 1 = 105$

So $\mathbf{(1101001)_2 = (105)_{10}}$

Ex. Convert (110101.1101)2 to decimal.

$(1*2^5) + (1*2^4) + (0*2^3) + (1*2^2) + (0*2^1) + (1*2^0) + (1*2^{-1}) + (1*2^{-2}) + (0*2^{-3}) + (1*2^{-4})$

$32 + 16 + 0 + 4 + 0 + 1 + 0.5 + 0.25 + 0 + 0.0625 = 53.5625$

Binary Addition:

0 + 0 = 0
1 + 0 = 1
0 + 1 = 1
1 + 1 = 0 *(carry 1)*
Ex. 101 + 111 = 1100
Ex. 1101 + 111 = 10100

Binary Subtraction:

0 – 0 = 0
1 – 0 = 1
0 – 1 = 1 *(borrow 1 from left column)*
1 – 1 = 0
Ex. 101011 – 100101 = 000110
Ex. 1111 – 1011 = 0100